programming of the matrix triangulation approach [4] could also be applied to the transformed lossless reflection factor functions yielding even greater accuracy. However, it appears that in practice the simple Richards' algorithm with remainder truncation applied to the appropriate lossless reflection factor data gives excellent results.

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Differential Phase Shift at Microwave Frequencies Using Planar Ferrites

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Abstract—**It is generally believed that planar ferrites are not useful in differential-phase-shift devices operated below resonance. A configuration of planar ferrite is suggested, which leads to appreciable differential phase shift on application of very small magnetic fields.**

The planar ferrite has an easy plane of magnetization. A small external magnetic field directed along some axis (z axis) in this plane saturates the sample magnetically. The nonunit diagonal components μ_{xx} and μ_{yy} of the permeability tensor are unequal [1]. The planar ferrites, when used in devices operating in Q and K bands, need the application of very small magnetic fields.

Bady [1, p. 59] stated that "... planar ferrites are not as desirable as isotropic ferrites ..." in operation below resonance because the change in susceptibility is small. In his case the easy plane yz (see Fig. 1) was in the plane of the slab and contained the direction of propagation y. It is shown in the following discussion that much higher differential phase shift can be achieved by orienting the easy plane normal to the direction of propagation.

Consider a thin slab of transversely magnetized ferrite, kept in a waveguide supporting only the dominant mode propagating along the y axis. The direction of magnetization is along the z axis. The broad face of the slab is normal to the x axis. The width of the guide along x axis is L, the distance of the slab from the sidewall at $x = 0$ is a and the slab thickness is δ. With μ_{xx} not necessarily the same as μ_{yy}, an approximate expression

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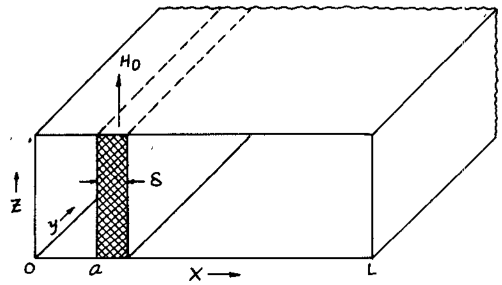

Fig. 1. A single planar ferrite slab in a rectangular waveguide of internal width L.

for the differential phase shift can be obtained [2] as follows:

$$\beta_+ - \beta_- = -\frac{2\pi\delta}{L^2} \cdot \frac{K}{\mu_{xx}} \sin 2\pi a/L \tag{1}$$

where β_+ and β_- are the propagation constants for the forward and the reverse propagation, K is the modulus of the off-diagonal element of the permeability tensor, and $\delta/L \simeq 0.01$ or so.

For a differential phase shifter using an isotropic ferrite slab (case 1), $\mu_{xx} = \mu_{yy}$ and (1) takes the familiar form [2]

$$\beta_+ - \beta_- = -\frac{2\pi\delta}{L^2} \cdot \frac{\omega \cdot 4\pi M_0/\gamma}{H_0(H_0 + 4\pi M_0) - \omega^2/\gamma^2} \sin 2\pi a/L \tag{2}$$

where ω, M_0, and H_0 are the operating frequency, saturation magnetization, and the applied magnetic field, respectively. γ is the gyromagnetic ratio to be taken as 1.76×10^7 rad/s.

For a differential phase shifter with the plane yz as the easy plane (case 2)

$$\beta_+ - \beta_- = -\frac{2\pi\delta}{L^2} \cdot \frac{\omega \cdot 4\pi M_0/\gamma}{H_0(H_0 + H_a + 4\pi M_0) - \omega^2/\gamma^2} \sin 2\pi a/L \tag{3}$$

where H_a is the anisotropy field and H_0 is the applied field, not necessarily the same as that in (2).

The ferrite phase shifters are normally operated below resonance to avoid attenuation. When the applied field is small, the first term in the denominators of (2) and (3) can be neglected in comparison with the second, and thus insignificant differential phase shifts result in both cases at relatively high frequencies.

The performance of a phase shifter using planar ferrite can be improved very much by orienting the easy plane normal to the direction of propagation (case 3). The expression for the differential phase shift assumes the following form:

$$\beta_+ - \beta_- = -\frac{2\pi\delta}{L^2} \cdot \frac{\omega \cdot 4\pi M_0/\gamma}{(H_0 + 4\pi M_0)(H_0 + H_a) - \omega^2/\gamma^2} \sin 2\pi a/L. \tag{4}$$

When M_0 and H_a are chosen suitably, considerably large differential phase shift can be obtained even when H_0 is small.

Normalized differential phase shift for the three aforementioned cases has been plotted in Fig. 2. It is obvious that maximum differential phase shift is obtainable in case 3 for small magnetic fields. However, in case 3 one would like to avoid the region $H_0 > 0.5$ kOe to keep away from resonance.

One disadvantage of the configuration used in case 3 is an overdependence of differential phase shift on the material

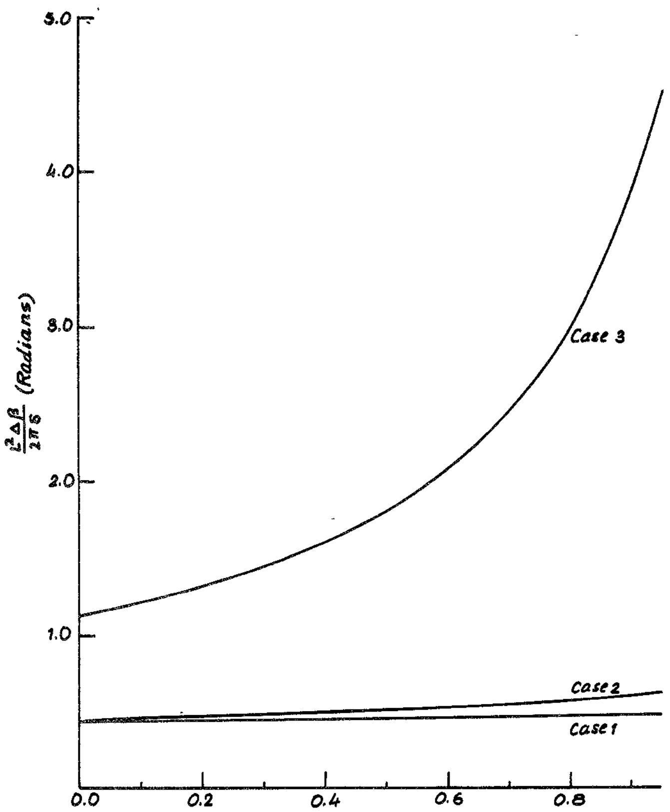

Fig. 2. Normalized differential phase shift versus magnetic field for different configurations. $\omega = 18$ GHz, $4\pi M_0 = 2.8$ kOe, $H_a = 9.0$ kOe (for Zh_2Y [3]).

properties M_0 and H_a. This limits the use of a particular slab in a narrow band of frequencies. Dielectric loading [4] may improve the band performance.

It is noteworthy that (1)–(4) are approximately valid only for very thin slabs. The rigorous derivation of differential phase shift and its applications are currently under investigation. The results will be published later.

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Electric-Field Distribution Along Finite Length Lossy Dielectric Slabs in Waveguide

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***Abstract*—A procedure is given to calculate the reflection and transmission coefficients of a full-height dielectric slab centered in a rectangular waveguide. The effects of loss and of finite length are included. The magnitude squared of the electric field along the slab is calculated in order to predict inhomogeneous heat input to the sample. These results are compared with experimental measurements on several materials and the pupae of *Tenebrio molitor*.**

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I. Introduction

The presence of standing waves in biological specimens, caused by internal nonuniformities, or by reflection from the boundaries of the specimen, can lead to localized heating. The distribution of the electromagnetic (EM) fields induced in such tissue has been modeled for planar, cylindrical, and spherical bodies under plane-wave or near-field illumination [1]–[8]. Experiments in which the biological specimen is placed in a waveguide for the purpose of irradiation have also been reported [9]–[11]. Field concentration effects occur in waveguides inhomogeneously loaded with a dielectric medium. If the dielectric is lossy, these effects can produce excess absorption of the microwave energy [12]–[14] in addition to the nonuniform absorption caused by the standing waves.

In this short paper, the biological specimen is modeled as a lossy dielectric slab of finite length inserted along the center line of a waveguide and calculations are presented for the following: 1) the magnitude of the electric field along the slab, 2) transmission and reflection coefficients, and 3) the percentage of incident power absorbed. The techniques and results to be presented here can also be applied to the calculation of power dissipation and heating in waveguide components such as attenuators and phase shifters.

II. Theory

Consider a slab of dielectric material of width D, length l, relative dielectric constant ε_r, and loss tangent $\tan\delta$, located along the center line of a rectangular waveguide of width a as shown in Fig. 1. The slab fills the height b of the waveguide. Assuming that a TE_{10} wave is incident from $-\infty$, the transverse components of the electric and magnetic fields can be expressed as a linear combination of higher order modes [15] in each of the three regions shown in the figure.

In Front of the Slab (Region I): $-\infty < z < 0$

$$E_y = f_1(x)e^{-jh_1 z} + \sum_{n=1,3,5\cdots}^{\infty} a_n f_n(x) e^{jh_n z} \tag{1a}$$

$$H_x = \frac{-f_1(x)}{Z_1} e^{-jh_1 z} + \sum_{n=1,3,5\cdots}^{\infty} \frac{a_n}{Z_n} f_n(x) e^{jh_n z}. \tag{1b}$$

In the Slab (Region II): $0 < z < l$

$$E_y = \sum_{m=1,3,5\cdots}^{\infty} a_m' g_m(x) e^{-\gamma_m z} + \sum_{m=1,3,5\cdots}^{\infty} b_m' g_m(x) e^{\gamma_m z}. \tag{2a}$$

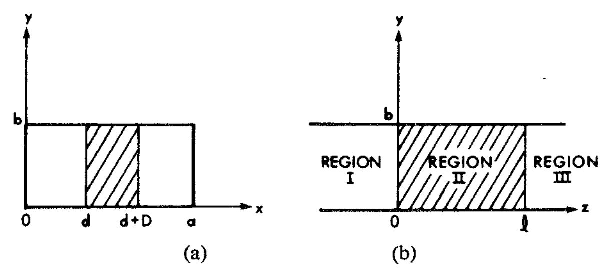

Fig. 1. Configuration of a finite length lossy dielectric slab located in a rectangular waveguide. (a) Cross-sectional view in a yx plane (i.e., down the guide) for which $0 < z < l$. (b) Cross-sectional view in a yz plane for which $d < x < d + D$.